Satisfaction Equilibrium

Definition, Learning Dynamics and Applications

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Alcatel Lucent Chair in Flexible Radio at Supelec

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Workshop on Algorithmic Game Theory: Dynamics and Convergence in Distributed Systems
AlgoGT 2011.
June, 20-21th 2011, Grenoble, France

Outline

Motivation

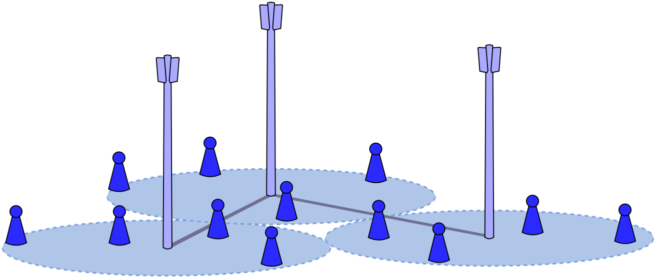
Satisfaction Equilibrium (SE)

Learning Satisfaction Equilibrium

Conclusions

Motivation

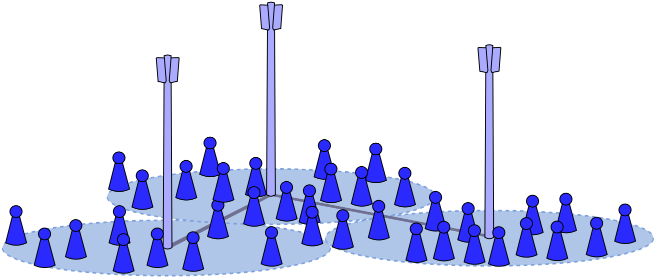
Clasical Cellular Systems



- Base Stations (BS) determines optimal Tx/Rx configurations.
- Terminals are not necessarily cognitive radios.

Motivation

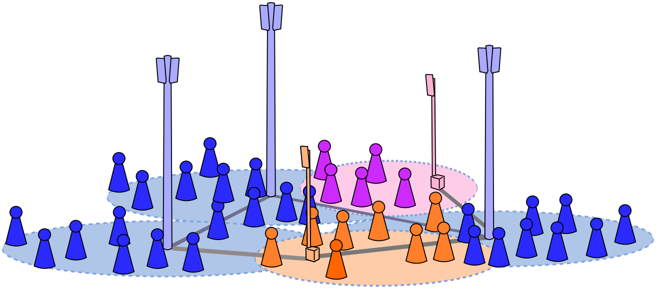
Classical Cellular Systems



- Explosion of number of communicating radio devices.
- Radio devices become more flexible: SDR, cognitive radio.

Motivation

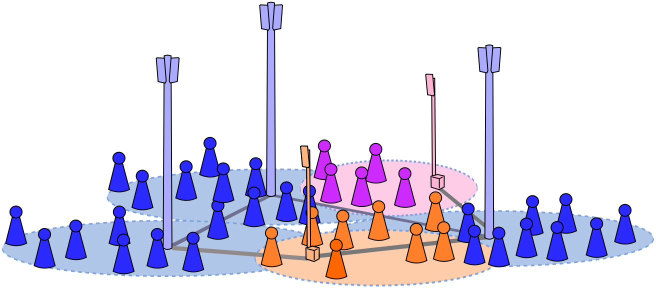
Today's Cellular Systems



- Need of additional infrastructure: femtocells, smallcells, relays, etc.
- Decentralized/uncoordinated approach.
- **How to provide QoS?**

Motivation

Modeling Decentralized Networks as Games



$$\mathcal{G} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right)$$

- **Players**: Set $\mathcal{K} = \{1, \ldots, K\}$ of transmitters and/or receivers.
- **Actions**: Set $\mathcal{A}_k$ of transmission/receive configurations of player $k \in \mathcal{K}$.
- **Utility function**: $u_k : \mathcal{A}_1 \times \ldots \times \mathcal{A}_K \to \mathbb{R}$ is a performance metric of player k.

Nash Equilibrium

J. F. Nash, “Equilibrium points in n-person games”, Proceedings of the National Academy of Sciences of the United States of America, vol. 36, no. 1, pp. 48-49, 1950.

Definition (Nash Equilibrium (Nash-1950))

An action profile $\boldsymbol{a}^* \in \mathcal{A}$ *is a NE of* $\mathcal{G}$*, if* $\forall k \in \mathcal{K}$ *and* $\forall \boldsymbol{a}'_k \in \mathcal{A}_k$*,*

$$u_k(a^*_k, \boldsymbol{a}^*_{-k}) \geqslant u_k(a'_k, \boldsymbol{a}^*_{-k}). \quad (1)$$

- Each player’s action is **optimal** w.r.t. the actions of all the other players.
- What if it is required that $u_k(a^*_k, \boldsymbol{a}^*_{-k}) > \Gamma_k$?.

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Generalized Nash Equilibrium

G. Debreu, "A social equilibrium existence theorem," Proceedings of the National Academy of Sciences of the United States of America, vol. 38, no. 10, pp. 886-893, October 1952.

- The set of each player's actions is **constrained**.

$$f_k : \mathcal{A}_{-k} \to 2^{\mathcal{A}_k} \tag{2}$$

- Consider the **extended version** of the normal form:

$$\tilde{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right). \tag{3}$$

Definition (Generalized NE (Debreu-1952))

An action profile $\boldsymbol{a}^* \in \mathcal{A}$ is a generalized Nash equilibrium (GNE) of $\tilde{\mathcal{G}}$, if $\forall k \in \mathcal{K}$ and $\forall a_k \in f_k\left(\boldsymbol{a}^*_{-k}\right)$,

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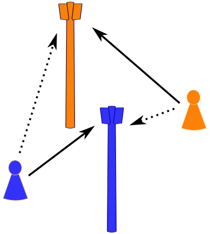
Example 1

2 Tx-Rx Pairs Sharing 1 freq. band:

- **Minimum rate** required: Γ_k bps.
- N_k **Transmit Power Levels**.



$$\tilde{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right)$$

- Players $\mathcal{K} = \{1, 2\}$.
- Actions: **Power Control**

$$\mathcal{A}_k = \left\{p_k^{(1)}, \ldots, p_k^{(N_k)}\right\}.$$

- Utility Function

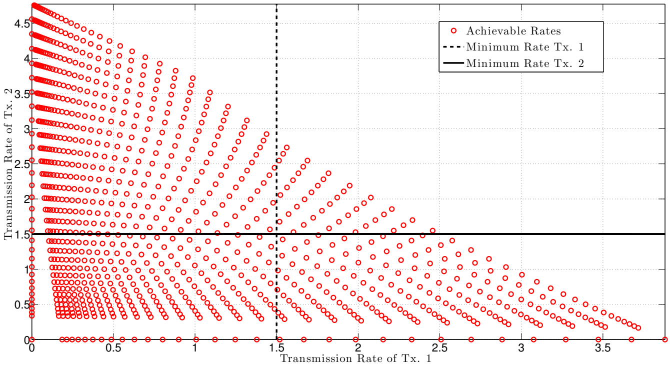
$$u_k(p_k, p_{-k}) = \log_2\left(1 + \frac{p_k |h_{k,k}|^2}{\sigma_k^2 + \sum_{j\neq k}^{K} p_{-k} |h_{k,-k}|^2}\right) \text{[bps]},$$

- Constraints:

$$f_k(p_{-k}) = \left\{p_k \in \mathcal{A}_k : u_k\left(p_k, p_{-k}\right) \geqslant \Gamma_k\right\}$$

Example 1

Achievable Rates + Minimum Rates



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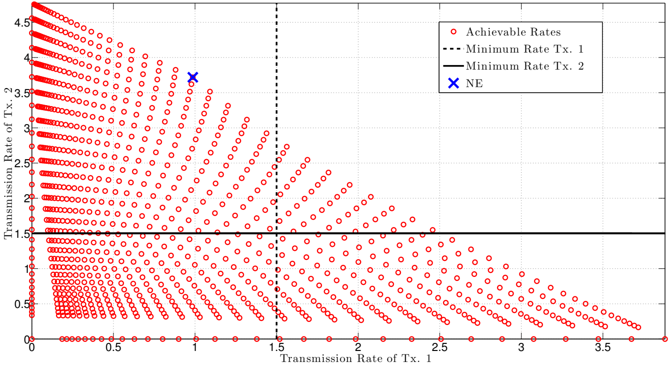
Achievable Rates + Minimum Rates + NE



At the NE, **only player** 2 **is satisfied**.

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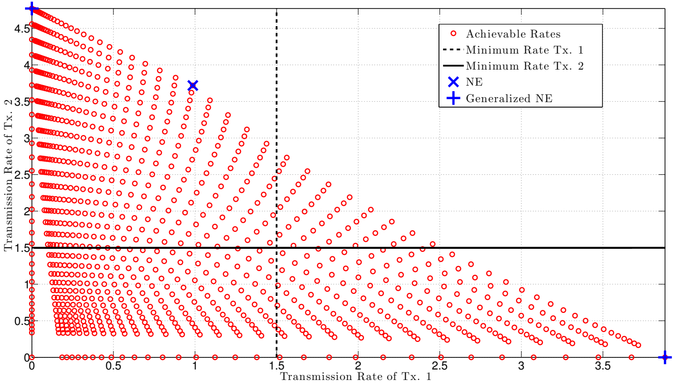
Achievable Rates + Minimum Rates + NE + GNE



At a GNE, **either player** 1 **or player** 2 are satisfied.

Our Contributions

A General Framework for Quality-Of-Service Provisioning in Decentralized Networks.

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- Consider the game in *satisfaction form*:

$$\bar{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right) \tag{6}$$

- The notion of **utility maximization** or **cost minimization** does not exist.

Definition (Satisfaction Equilibrium)

An action profile $\boldsymbol{a}^+$ *is a satisfaction equilibrium of* $\tilde{\mathcal{G}}$ *if* $\forall k \in \mathcal{K}$,

$$a_k^+ \in f_k\left(\boldsymbol{a}_k^+\right). \tag{7}$$

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SE is a generalization of NE

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- Consider the games in NF and SF

$$\mathcal{G} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right)$$

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- Let the correspondence f_k be defined as follows:

$$f_k(\boldsymbol{a}_{-k}) = \mathrm{BR}_k\left(\boldsymbol{a}_{-k}\right) = \arg\max_{a_k\in\mathcal{A}_k} u_k\left(a_k, \boldsymbol{a}_{-k}\right) \quad (8)$$

- Then, **the set of SE of $\bar{\mathcal{G}}$ and the set of NE of $\mathcal{G}$ coincide.**

Let $\boldsymbol{a} \in \mathcal{A}$ be a NE. Then,

$$\boldsymbol{a} \in \mathrm{BR}(\boldsymbol{a}), \quad (9)$$

where, $\mathrm{BR}\left(\boldsymbol{a}\right) = \mathrm{BR}_1\left(\boldsymbol{a}_{-1}\right) \times \ldots \times \mathrm{BR}_K\left(\boldsymbol{a}_{-K}\right)$.

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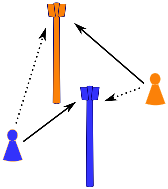
Example 2

2 Tx-Rx Pairs sharing 2 freq. band:

- **Minimum rate** required: Γ_k bps.
- N_k **Transmit Power Levels**.



- Players $\mathcal{K} = \{1, 2\}$.
- Actions: **Channel Selection**

$$\mathcal{A}_k = \left\{ (p_{k,\max}, 0), (0, p_{k,\max}) \right\}.$$

- Constraints:

$$\hat{u}_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k}) = \sum_{s=1}^{S} \log_2 \left(1 + \frac{p_{k,s} |h_{k,k}^{(s)}|^2}{\sigma_k^2 + \sum_{j \neq k}^{K} p_{-k,s} |h_{k,-k}^{(s)}|^2} \right) \text{[bps]},$$

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- Utility Function

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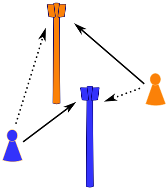

$P_1 \backslash P_2$	$A_2^{(1)}$	$A_2^{(2)}$
$A_1^{(1)}$	$(0, 0)$	$(1, 1)$
$A_1^{(2)}$	$(1, 0)$	$(0, 0)$

Figure: Player 1 chooses rows and player 2 chooses columns.

- 2 NE in $\mathcal{G}$: $(A_1^{(2)}, A_2^{(1)})$ and $(A_1^{(1)}, A_2^{(2)})$,
- 1 SE in $\bar{\mathcal{G}}$: $(A_1^{(1)}, A_2^{(2)})$.

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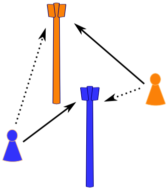

- In these particular games,

$$\mathcal{A}_{\mathrm{SE}} \subseteq \mathcal{A}_{\mathrm{NE}} \subseteq \mathcal{A}. \tag{10}$$

- At NE, **all players might not be satisfied**.
- At SE, if it exists, **all players are satisfied**.

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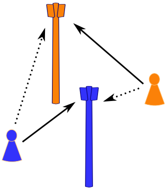

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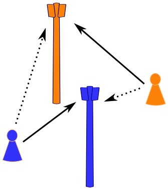

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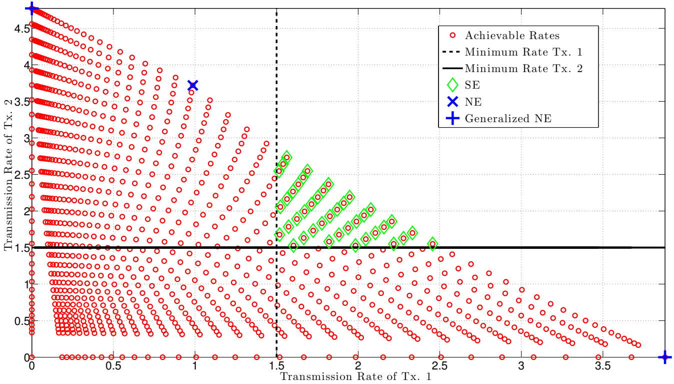
Achievable Rates + Minimum Rates + NE + GNE + SE



Neither the NE nor GNE properly model the QoS provisioning problem.

Does a SE always exist?

Let the correspondence $F : \mathcal{A} \rightarrow 2^{\mathcal{A}}$ be defined as follows:

$$F(\boldsymbol{a}) = f_1(\boldsymbol{a}_{-1}) \times \ldots \times f_K(\boldsymbol{a}_{-K}).$$

Then, an SE exists if and only if

$$\exists \boldsymbol{a} \in \mathcal{A} : \boldsymbol{a} \in F(\boldsymbol{a}). \quad (11)$$

This formulation is a **fixed point inclusion**!

Proposition (Existence of the SE (Kakutani-1941))

*In the game $\bar{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right)$, let the set of actions $\mathcal{A}$ be a **non-empty, convex and compact set**. Let also the correspondence $F(\boldsymbol{a})$ have a **closed graph** and be **non-empty** and **convex** in the set of actions $\mathcal{A}$. Then, the game $\bar{\mathcal{G}}$ has at least one SE.*

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In the game $\bar{\mathcal{G}} = \left(\mathcal{K}, \left\{\mathcal{A}_k\right\}_{k \in \mathcal{K}}, \left\{f_k\right\}_{k \in \mathcal{K}}\right)$, let the set of actions $\mathcal{A}$ be a ***non-empty, convex and compact set****. Let also the correspondence $F(\boldsymbol{a})$ have a* ***closed graph*** *and be* ***non-empty*** *and* ***convex*** *in the set of actions $\mathcal{A}$. Then, the game $\bar{\mathcal{G}}$ has at least one SE.*

Existence of the SE

Many results from Fixed Point Inclusions are available:

- S. Kakutani, “A generalization of Brouwer’s fixed point theorem”, Duke Mathematical Journal, vol. 8, pp. 457–459, 1941.
- B. Knaster and A. Tarski, “Un théorème sur les fonctions d’ensembles,” Ann. Soc. Polon. Math., vol. 6, pp. 133–134, 1928
- K. C. Border, Fixed Point Theorems with Applications to Economics and Game Theory. New York, NY, USA: Cambridge University Press, 1985.

Uniqueness of the SE

Perlaza, S. M. and Tembine, H. and Lasaulce, S. and Debbah, M., "**A General Framework for Quality-Of-Service Provisioning in Decentralized Networks**". Submitted to the IEEE Journal in Selected Topics in Signal Processing. Special Issue in Game Theory for Signal Processing. 2011

- A SE is in general **not unique**.
- Which action to play to be satisfied? (**Effort for Satisfaction**)
- How to select a SE? (**Efficient SE**)

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Efficient Satisfaction Equilibrium

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- The **set of ESE** of the game

$$\bar{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right),$$

  w.r.t. the cost functions $c_1, \ldots, c_K$, **coincides with the set of GNE** of the game

$$\tilde{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{c_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right).$$

- The game $\tilde{\mathcal{G}}$ is an **exact constrained potential game** (ECPG).

$$\text{Potential function:} \quad \phi(\boldsymbol{a}) = \sum_{k=1}^{K} c_k(a_k). \tag{12}$$

- ECPGs **do not have the same properties** than Potential Games.
- **Existence**, **Uniqueness** and **convergence of the best response dynamics** are constrained.

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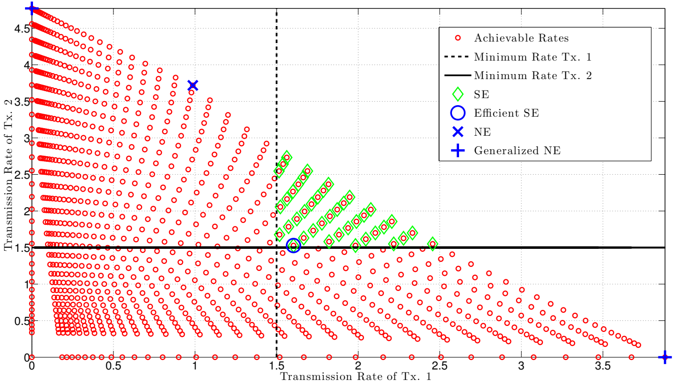
Example 1

Achievable Rates + Minimum Rates + NE + GNE + SE + ESE

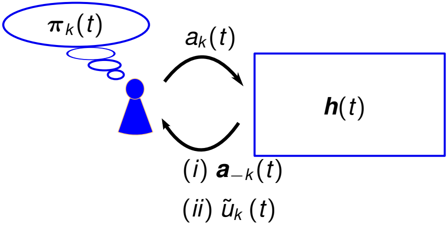


At ESE all players are satisfied by doing a **minimum effort**.

Learning Satisfaction Equilibria

Learning Iterative Steps:

- **Choose** action $a_k(t) \sim \pi_k(t)$.
- **Observe** game outcome, e.g.,
  $\boldsymbol{a}_{-k}(t)$
  $u_k(a_k(t), \boldsymbol{a}_{-k}(t))$.
- **Improve** $\pi_k(t+1)$.



Thus, we can expect that: $\forall k \in \mathcal{K}$,

$$a_k(t) \quad \xrightarrow{t \to \infty} \quad \boldsymbol{a}_k^*, \tag{13}$$

where, $\boldsymbol{a}^* = (\boldsymbol{a}_1^*, \ldots, \boldsymbol{a}_K^*)$ is a SE action profile.

Learning Satisfaction Equilibria

S. Ross and B. Chaib-draa, "Learning to play a satisfaction equilibrium." in Workshop on Evolutionary Models of Collaboration, 2007.

1. Observation at iteration t:

$$\tilde{u}_k(t) = \begin{cases} 1 & \text{if} \quad a_k(t) \in f_k\left(\boldsymbol{a}_{-k}(t)\right) \\ 0 & \text{otherwise.} \end{cases}$$

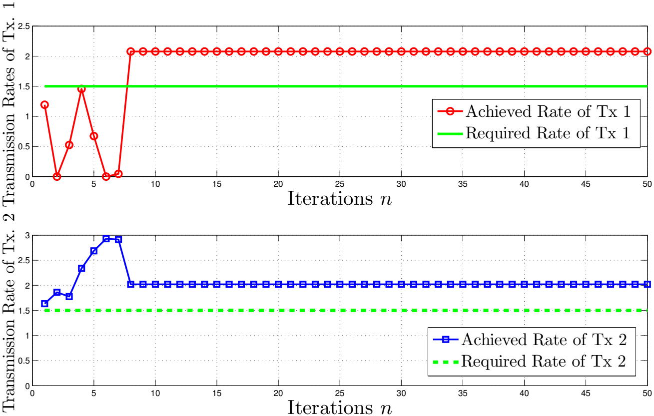
2. Action at iteration t:

$$a_k(t) = \begin{cases} a_k(t-1) & \text{if} \quad \tilde{v}_k(t) = 1 \\ a_k(t) \sim \boldsymbol{\pi}_k(t) & \text{otherwise.} \end{cases} \tag{14}$$

3. Strategy at iteration t: $\boldsymbol{\pi}_k(t) = \left(\pi_{k,A_k^{(1)}}(t), \ldots, \pi_{k,A_k^{(N_k)}}(t)\right)$,

$$\forall n_k \in \{1, \ldots, N_k\}, \quad \hat{\pi}_{k,A_k^{(n_k)}}(n) = \frac{1}{N_k}. \tag{15}$$

Example 1



Transmission Rates of Tx. 1
Iterations n
Achieved Rate of Tx 1
Required Rate of Tx 1
Transmission Rate of Tx. 2
Iterations n
Achieved Rate of Tx 2
Required Rate of Tx 2

Learning Satisfaction Equilibria

Definition (Clipping Action)

A player k is said to have a clipping action $a_k \in \mathcal{A}_k$ in the game $\bar{\mathcal{G}} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{f_k\}_{k\in\mathcal{K}}\right)$ if

$$\forall \boldsymbol{a}_{-k} \in \mathcal{A}_{-k}, \quad a_k \in f_k\left(\boldsymbol{a}_{-k}\right). \tag{16}$$

Proposition

Let $a_k^* \in \mathcal{A}_k$ be a clipping strategy for player k. Then, if there exists a player $j \in \mathcal{K} \setminus \{k\}$, and an action profile $(a_k^*, \cdot) \in \mathcal{A}_{-j}$ for which $f_j\left(a_k^*, \cdot\right) = \emptyset$. Then, the behavioral rule in (14) does not converge to a SE with strictly positive probability.

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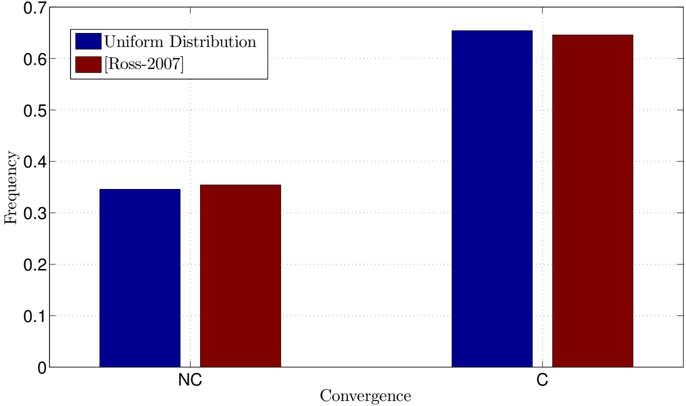
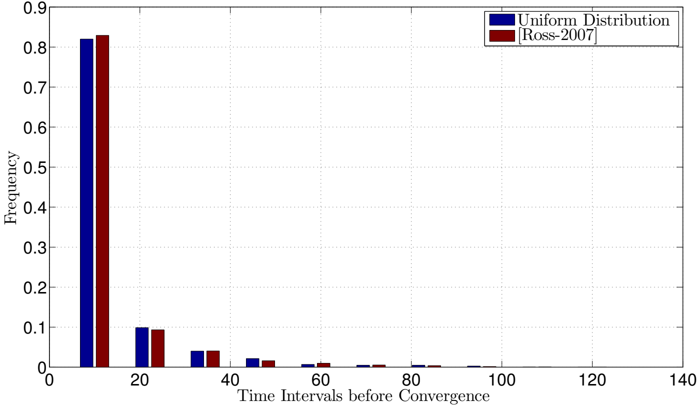

Uniform Distribution
[Ross-2007]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Frequency
NC
C
Convergence

Example 1


Uniform Distribution
[Ross-2007]
Frequency
Time Intervals before Convergence
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140

Conclusions

- A suitable equilibrium for **QoS provisioning in small cells** exists.
- Learning Dynamics for **achieving SE** have been presented.
- Several **convergence issues** remain to be solved in SE.
- Learning Dynamics for **achieving ESE** are still unknown.

Further Work ...

- What if no SE exists? ϵ-**SE**.
- How to overcome a Clipping Action? **Trial and Error Learning.**
- How to achieve an ESE?

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